DISCUSSION OF J. MIYAZAWA

1. Problems for first session

(1) Consider $X = \mathbb{CP}^2 \setminus \text{int } D^4$ and the trefoil $K = T_{2,3}$ in its boundary S^3. Show that K bounds a disk in X (i.e., it is slice in X).
(2) (Trace embedding lemma) Show that the knot $K \subset S^3 \subset D^4$ is slice (i.e. bounds a smooth disk in D^4) if and only if the 0-trace $X(K)$ (which is the 4-disk D^4 together with a 0-framed 2-handle attached along K) embeds into S^4.
(3) Show that a homology class α in $H_2(S^2 \times S^2; \mathbb{Z})$ (or in $H_2(\mathbb{CP}^2 \# \overline{\mathbb{CP}^2}; \mathbb{Z})$) with $\alpha \cdot \alpha = 0$ can be represented by a sphere.
(4) Show that a smooth complex curve of degree d in $\mathbb{CP}^2$ (so $d > 0$) has genus $\frac{1}{2}(d-1)(d-2)$.
(5) Show that the map $SU(2) \to SO(3)$ given by

$$h \mapsto \phi_h(x) = h^{-1} x h$$

viewing $SU(2)$ as unit quaternions and $x \in \text{Im } \mathbb{H}$ imaginary quaternion.
(6) The map $Spin^c(4) = SU(2) \times SU(2) \times S^1 / \{\pm(1,1,1)\} \to SO(4)$ given by (q_+, q_-, λ) acting on $\phi \in \mathbb{H}$ by $q_+ \phi q_-^-$ is the nontrivial S^1-bundle over $SO(4)$.